

Phenomenological theory of superconductivity near domain walls in ferromagnets

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We develop a phenomenological model of superconductivity near a domain wall in a ferromagnet. In addition to the electromagnetic interaction of the order parameter with the ferromagnetic magnetization, we take into account the possibility of a local enhancement or suppression of superconducting pairing in the vicinity of the wall, and also a non-perfect transparency of the wall to electrons. It is found that the critical temperature of superconductivity near the domain wall might be substantially higher than in the bulk.

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I. INTRODUCTION

The recent discovery of superconductivity co-existing with ferromagnetism in UGe₂ [1], ZrZn₂ [2], and URhGe [3] have revived the long-standing interest to the interplay of the two phenomena. Although the microscopic mechanism responsible for superconducting pairing in the presence of a ferromagnetic background is still subject to considerable controversy, some useful information about the properties of these systems can be obtained from a phenomenological approach, based on general symmetry arguments and the Ginzburg-Landau (GL) equations, see e.g. Ref. [4]. A symmetry-based description of ferromagnetic (FM) superconductors has been developed in Refs. [5, 6, 7, 8, 9, 10, 11].

In contrast to the non-magnetic case, a phenomenological theory of superconductivity in the presence of ferromagnetism should include the effects of internal magnetic induction \mathbf{B} created by the macroscopic FM magnetization \mathbf{M} . That the coupling of the orbital motion of electron charges with \mathbf{B} would suppress superconductivity even in zero external field was pointed out by Ginzburg [12], who argued that superconductivity could not exist in any of the FM materials known at that time. While valid in a single-domain ferromagnet, his argument should be taken with care if \mathbf{B} is non-uniform. In order to minimize the magnetic field energy, it is energetically favorable for a macroscopic ferromagnet to break up into domains of size L with opposite directions of the magnetization \mathbf{M} , separated by domain walls (DWs) of thickness l . Due to the spatial variation of the internal field, it is expected that the suppression of superconductivity in the vicinity of a DW is not as strong as in the bulk, as was first hypothesized in Ref. [13]. The quantitative analysis shows [14] that the superconducting critical temperature near a DW is indeed higher than the bulk critical temperature deep inside the domain. The reason is that the effective GL potential near the wall has the same double-well shape as near a superconductor-insulator interface, leading to a local enhancement of superconductivity similar to the surface sheath effect [15]. The same model was recently extended in Ref. [16] to calculate the critical temperature of the superconducting DW in an external magnetic field.

In all previous treatments of the superconducting DW problem it was assumed that the superconducting coupling constant is the same throughout the system. However, it can be expected that, whatever the microscopic pairing mechanism, the conditions for the formation of the Cooper pairs near the DW and far from it are different. The FM magnetization affects the electrons via the orbital interaction of the electron charges with the induction \mathbf{B} , the exchange band splitting, and also the Zeeman coupling of the electron spins with \mathbf{B} . Because of the spatial variation of both the exchange field and the internal induction, all three mechanisms could change both the single-electron spectrum and the electron-electron interactions in the vicinity of the DW, compared to the bulk of the domains. In particular, the strength of the Cooper pairing could be locally enhanced or suppressed near the wall. Also, the DW creates a potential barrier for electrons travelling from one domain to the other. The possibility of a non-perfect transparency of the barrier should therefore be included in the theory.

We assume that the thickness of the wall is small compared with the length scales of superconductivity: $l \ll \xi_0 \ll \lambda_L$, where ξ_0 is the coherence length and λ_L is the London penetration depth. Then, in addition to the usual (i.e. single-domain) terms, the general phenomenological GL functional includes interface contributions localized near the DW, which describe the variation of the pairing strength and a finite transparency of the DW for electrons. These additional terms affect the boundary conditions for the superconducting order parameter at the wall [17], and the problem then becomes formally similar to that of the localized superconductivity near a planar crystalline defect, which has been extensively studied in the context of the twinning plane superconductivity in high- T_c cuprates and other superconductors [18, 19, 20].

The purpose of the present article is to study the phase diagram of the localized DW superconductivity in a general phenomenological model. The article is organized as follows: In Sec. II, we formulate our model and derive the GL equations with appropriate boundary conditions. In Sec. III, we calculate the critical temperature of the DW superconductivity as a function of an external field. The evolution of the shape of the superconducting nucleus in

the non-linear regime below the critical temperature is studied in Sec. IV.

II. THE MODEL

We assume that the Curie temperature T_{FM} is higher than the critical temperature T_{SC} and treat the system in the mean-field approximation, so the superconductivity appears in the presence of a static FM background. According to Refs. [7, 8, 9, 10], the order parameter in a FM superconductor transforms according to one of the irreducible co-representations of the magnetic symmetry group in the normal state. Here we assume that the order parameter has one component $\psi(\mathbf{r})$, which is always the case in, e.g., ZrZn₂, assuming that only one of the exchange-split bands is superconducting (in principle, the pair scattering can induce order parameters of the same symmetry on different sheets of the Fermi surface, leading to a more complicated form of the free energy functional than the one used below). The spin quantization axis is chosen parallel to $\mathbf{M}(\mathbf{r})$, so that the pairing in both domains occurs in, say, spin-up channel. Thus the superconducting order parameters are $\Delta_+(\mathbf{k}, \mathbf{r}) = \Delta_{\uparrow\uparrow}^{(+)}(\mathbf{k}, \mathbf{r}) = \psi_+(\mathbf{r})\phi(\mathbf{k})$, $\Delta_-(\mathbf{k}, \mathbf{r}) = \Delta_{\uparrow\downarrow}^{(-)}(\mathbf{k}, \mathbf{r}) = \psi_-(\mathbf{r})\phi(\mathbf{k})$, where $\phi(\mathbf{k})$ is the basis function of the chosen one-dimensional co-representation.

The bulk critical temperature is the same on both sides of the wall and equal to T_{c0} . We focus here on the case $l \ll \xi_0 \ll L$, where ξ_0 is the superconducting coherence length, which allows us to consider a single wall separating two domains with opposite directions of magnetization. The GL free energy density can be represented as a sum of the bulk contributions from the two domains, and the interface contribution due to the presence of the DW: $F = F_+ + F_- + F_{DW}$. Choosing the DW to coincide with the plane $x = 0$, we have

$$F_{\pm} = a(T - T_{c0})|\psi_{\pm}|^2 + K|\mathbf{D}\psi_{\pm}|^2 + \frac{1}{2}\beta|\psi_{\pm}|^4, \quad (1)$$

where $\psi_+ = \psi(x > 0)$ and $\psi_- = \psi(x < 0)$ are the order parameters in the right and left domains respectively, $D_i = -i\nabla_i + (2\pi/\Phi_0)A_i(\mathbf{r})$, $\nabla \times \mathbf{A} = \mathbf{B}$, $\Phi_0 = \pi\hbar c/e$ is the magnetic flux quantum (the electron charge is equal to $-e$). The internal magnetic induction is given by $\mathbf{B} = 4\pi\mathbf{M} + \mathbf{H}$, where \mathbf{H} is an external field. Since the DW thickness is much smaller than ξ_0 , we can approximate the non-uniform magnetization by a step-like function: $\mathbf{M}(\mathbf{r}) = (0, 0, M_0 \text{ sign } x)$. If the external field is directed along the z axis, we have $\mathbf{A}(\mathbf{r}) = (0, B_0|x| + Hx, 0)$, where $B_0 = 4\pi M_0$ (we assume $M_0 > 0$). For simplicity, we neglect the anisotropy of the effective mass tensor in the gradient terms.

The DW contribution, which is localized near $x = 0$, can be written in the following general form:

$$\begin{aligned} F_{DW} = & [\gamma_1(|\psi_+|^2 + |\psi_-|^2) - \gamma_2(\psi_+^*\psi_- + \psi_-^*\psi_+)] \\ & + i\gamma_3(\psi_+^*\psi_- - \psi_-^*\psi_+)\delta(x), \end{aligned} \quad (2)$$

with real coefficients γ_i . We keep only the terms quadratic in ψ_{\pm} , which are needed to obtain linear boundary conditions at $x = 0$, see below. The expression (2) is consistent with the symmetry of the system: F_{DW} has to be real and invariant with respect to time reversal ($\psi_{\pm} \rightarrow \psi_{\pm}^*$) accompanied by the interchange of the two domains ($+\leftrightarrow-$). The parameters of the GL functional depend on the domain magnetization \mathbf{M} . In particular, in the limit $\mathbf{M} \rightarrow 0$ (no domain wall), all γ_i should vanish. While the first two terms in Eq. (2) are similar to those discussed in the context of the localized superconductivity at planar crystalline defects [19], the last one is unique to the present problem because it requires time-reversal symmetry breaking. It turns out however that we can choose $\gamma_3 = 0$ without any loss of generality, because the second and third terms in Eq. (2) can be combined together by rotating the order parameter phases separately in each domain: $\psi_{\pm} \rightarrow e^{\mp i\theta/2}\psi_{\pm}$, where $\theta = \arg(\gamma_3/\gamma_2)$. For the same reason, the sign of γ_2 is not important, in particular one can always choose it to be positive: if $\gamma_2 < 0$, then the gauge transformation $\psi_+ \rightarrow \psi_+$, $\psi_- \rightarrow e^{i\pi}\psi_-$ changes $\gamma_2 \rightarrow -\gamma_2$.

In our phenomenological model we allow for the order parameter to be discontinuous across the domain wall, the rationale being that the DW creates a potential barrier for electrons and therefore may behave similar to an extended Josephson junction. The interface terms (2) then translate into the linear boundary conditions for ψ_{\pm} , see e.g. Ref. [17]. The magnitude of the coupling between the domains, described by the γ_2 term, depends on the transmission coefficient of the DW for electrons, which in turn is determined by the thickness and the structure of the wall. In the limit of zero transparency, $\gamma_2 = 0$ and the domains are completely decoupled. The continuity of the order parameter is restored in the limit $\gamma_1 = \gamma_2 \rightarrow +\infty$, when the interface terms can be written as $\gamma_1|\psi_+ - \psi_-|^2$, which imposes the condition $\psi_+(x = 0) = \psi_-(x = 0)$ [and also $\psi'_+(x = 0) = \psi'_-(x = 0)$, see below]. If $\gamma_1 = \gamma_2 + \delta\gamma$, with $\gamma_{1,2} \rightarrow +\infty$ and a finite $\delta\gamma$, then the order parameter is continuous, and $F_{DW} = \delta\gamma|\psi|^2\delta(x)$. Depending on the sign of $\delta\gamma$, this term describes either a local enhancement ($\delta\gamma < 0$) or suppression ($\delta\gamma > 0$) of superconductivity near the DW.

Far from the wall, the magnetic induction is uniform and the field dependence of the superconducting critical temperature is given by the standard upper critical field expression $T_{c,bulk} = T_{c0} - 2\pi KB_0/\Phi_0a$ (one should keep in mind that T_{c0} itself can be a function of the magnetization). Thus the superconductivity in the bulk is suppressed by the internal induction B_0 even in the absence of the external field [12]. At $T < T_{c,bulk}$, both domains are in the mixed state.

Let us now calculate T_c in the vicinity of the wall. We assume that the phase transition into the superconducting state is of second order at all fields. To find the phase diagram in the external field \mathbf{H} , we use the linearized GL equations obtained from Eq. (1), with the

boundary conditions supplied by the interface terms (2). The order parameter has the form $\psi_{\pm}(\mathbf{r}) = e^{iqy}f_{\pm}(x)$, where the functions $f_{\pm}(x)$ satisfy the equations

$$-K \frac{d^2 f_{\pm}}{dx^2} + K \left[\frac{2\pi}{\Phi_0} (H \pm B_0)x + q \right]^2 f_{\pm} + a(T - T_{c0})f_{\pm} = 0, \quad (3)$$

supplemented by the boundary conditions at $x = 0$:

$$\begin{cases} K \frac{df_+}{dx} = \gamma_1 f_+ - \gamma_2 f_-, \\ K \frac{df_-}{dx} = \gamma_2 f_+ - \gamma_1 f_-. \end{cases} \quad (4)$$

To simplify the notations, it is convenient to use dimensionless variables: $\tilde{x} = x/l_B$, $\tilde{q} = ql_B$, and $\tilde{\gamma}_i = \gamma_i l_B/K$, where $l_B = \sqrt{\Phi_0/2\pi B_0}$. Omitting the tildas, the GL equations take the form

$$-f''_{\pm} + [(1 \pm h)x \pm q]^2 f_{\pm} + (\tau - 1)f_{\pm} = 0, \quad (5)$$

where $h = H/B_0 \geq 0$ is the dimensionless external magnetic field, and $\tau = 1 + a(T - T_{c0})\Phi_0/2\pi B_0 K$ is the dimensionless temperature. It is easy to see that the zero-field bulk critical temperature $T_{c,bulk}$ corresponds to $\tau = 0$, while T_{c0} corresponds to $\tau = 1$. The boundary conditions (4) become

$$f'_{\pm} \mp \gamma_1 f_{\pm} \pm \gamma_2 f_{\mp} = 0. \quad (6)$$

As seen from Eqs. (5), the effective GL potential $V(x)$ is described by two parabolas matched at $x = 0$. We have to find the absolute minimum of the ground state energy $1 - \tau$ in this potential as a function of the parameter q . The solution of Eqs. (5) that does not diverge (in fact, vanishes) at $|x| \rightarrow \infty$, has the form

$$f_{\pm}(x) = C_{\pm} e^{-X_{\pm}^2/2} H_{\nu_{\pm}}(\pm X_{\pm}), \quad (7)$$

where C_{\pm} are some constants, $H_{\nu}(z)$ is the Hermite functions [21], and

$$\nu_{\pm} = \frac{1}{2} \left(\frac{1 - \tau}{|1 \pm h|} - 1 \right), \quad X_{\pm} = \sqrt{|1 \pm h|} \left(x \pm \frac{q}{1 \pm h} \right).$$

These expressions are singular at $h = \pm 1$, which requires special consideration, see below. Substituting $f_{\pm}(x)$ in the boundary conditions (6) and using the fact that $H'_{\nu}(z) = 2\nu H_{\nu-1}(z)$, we obtain the solutions of two types. First, there always exists a trivial bulk solution located deep inside the domain with lower induction, for which $q = +\infty$ or $-\infty$ and $1 - \tau = |1 \pm h|$, thus the critical temperature is simply $\tau_{c,bulk}(h) = 1 - \min |1 \pm h| = 1 - |1 - h|$ (at $h \geq 0$). Second, we can also have a non-trivial solution localized near the domain wall, which is sensitive to the values of γ_i . The corresponding equation for $\tau(h, q)$ is

$$\left[L_{\nu_+} \left(q \frac{\sqrt{|1+h|}}{1+h} \right) - \frac{\gamma_1}{\sqrt{|1+h|}} \right] \left[L_{\nu_-} \left(q \frac{\sqrt{|1-h|}}{1-h} \right) - \frac{\gamma_1}{\sqrt{|1-h|}} \right] = \frac{\gamma_2^2}{\sqrt{|1-h^2|}}. \quad (8)$$

Here we introduced $L_{\nu}(z) = 2\nu H_{\nu-1}(z)/H_{\nu}(z) - z$. The critical temperature $\tau_c(h)$ is given by the absolute maximum of $\tau(h, q)$ with respect to q at given h . Clearly $\tau_c(-h) = \tau_c(h)$ [this is consistent with Eq. (8) being invariant under $h \rightarrow -h$], therefore we assume $h \geq 0$.

As mentioned above, the case $h = 1$ requires special care, because of the singularities in Eq. (8). Physically, this corresponds to a complete compensation of the internal induction by the external field in the domain $x < 0$. While $f_+(x)$ in this case is still expressed in terms of the Hermite functions, $f_-(x)$ is given by a simple exponential. The equation for $\tau(h = 1, q)$ is

$$\left[\sqrt{2} L_{\nu} \left(\frac{q}{\sqrt{2}} \right) - \gamma_1 \right] \left(\sqrt{\tau - 1 + q^2} + \gamma_1 \right) + \gamma_2^2 = 0, \quad (9)$$

where $\nu = -(\tau + 1)/4$ [note that the critical temperature of the DW superconductivity is never below the bulk critical temperature, in particular $\tau_c(h = 1) \geq 1$].

III. PHASE DIAGRAM

A. Transparent DW

Before discussing the solution of Eq. (8) for arbitrary values of γ_1 and γ_2 , let us first look at some important particular cases. If the domain wall is completely transparent for the electrons, then $\gamma_1 = \gamma_2 + \delta\gamma$, $\gamma_{1,2} \rightarrow +\infty$, and $f_+ = f_-$. In this limit, Eq. (8) reduces to

$$\begin{aligned} & \sqrt{|1+h|} L_{\nu_+} \left(q \frac{\sqrt{|1+h|}}{1+h} \right) \\ & + \sqrt{|1-h|} L_{\nu_-} \left(q \frac{\sqrt{|1-h|}}{1-h} \right) = 2\delta\gamma. \end{aligned} \quad (10)$$

In the simplest case $\delta\gamma = 0$, corresponding to the absence of the local enhancement or suppression of superconductivity, the results of Ref. [16] are recovered from Eq.

(10). At zero external field we have $\nu_{\pm} = \nu = -\tau/2$, and Eq. (10) becomes $L_{\nu}(q) = 0$, whose numerical solution shows that the ground state energy $1 - \tau = 2\nu + 1$ reaches its minimum at $\nu = \nu_* \simeq -0.20$ and $q = q_* \simeq -0.77$. In terms of the critical temperature, this gives $\tau_c(h=0) = -2\nu_* \simeq 0.41$ and $T_c \simeq T_{c0} - 0.59(2\pi K B_0/\Phi_0 a) > T_{c,bulk}$. Thus, superconductivity can exist in the vicinity of the DW at a temperature higher than $T_{c,bulk}$. This effect has the same origin as the surface critical field H_{c3} [14]. At $h \neq 0$, the critical temperature first grows as a function of h , because the external field partially compensates the internal induction in one of the domains, and then finally starts to decrease at $h > 1$, see Fig. 1.

At $h = 0$ but $\delta\gamma \neq 0$, we have

$$L_{-\tau/2}(q) = \delta\gamma \quad (11)$$

If the superconductivity is locally suppressed, i.e. $\delta\gamma > 0$, then the critical temperature is always lower than at $\delta\gamma = 0$. In contrast, if the superconductivity is locally enhanced, i.e. $\delta\gamma < 0$, then τ_c is always higher than at $\delta\gamma = 0$.

The results of numerical solution of Eq. (10) are presented in Fig. 1. The bulk critical temperature $\tau_{c,bulk}(h)$ serves as the lower boundary for $\tau_c(h)$ in the limit of strong suppression of superconductivity near the DW. In the opposite limit of large negative $\delta\gamma$ (i.e. strong superconductivity enhancement), the influence of the variation of the internal induction is less pronounced, so that the phase diagram becomes similar to that of a superconducting plane defect in a non-magnetic crystal [18]. The order parameter is continuous and localized in the vicinity of the DW, see Figs. 2 and 3.

Another notable feature of the phase diagram is that, in contrast to the upper critical field in the bulk, $\tau_c(h)$ for the DW superconductivity is quadratic in field at $h \rightarrow 0$: $\tau_c(h) - \tau_c(h=0) = O(h^2)$. This happens because the localization of the order parameter near the DW makes it possible to treat h as a small perturbation, which cannot be done in the bulk case.

B. Decoupled domains

The opposite case $\gamma_1 = \gamma_2 = 0$ corresponds to completely decoupled domains with the order parameters obeying $f'_{\pm}(0) = 0$. As follows from Eq. (8), the critical temperature in this case is determined by the equation $L_{\nu_-}[q\sqrt{|1-h|}/(1-h)] = 0$ at $h \geq 0$, which gives $\tau_c(h) = 1 - (2\nu_* + 1)|1-h| \simeq 1 - 0.59|1-h|$, so we have an H_{c3} -type behavior at all fields.

C. General case

Let us now find the critical temperature and the shape of the superconducting nucleus at zero external field for arbitrary γ_1 and γ_2 . At $h = 0$ Eq. (8) yields two equations for the critical temperature: $L_{-\tau/2}(q) = \gamma_{\pm}$, where

$\gamma_{\pm} = \gamma_1 \pm \gamma_2$. Recalling that $\gamma_2 \geq 0$, the highest critical temperature is achieved when the right-hand side is $\gamma_- = \delta\gamma = \gamma_1 - \gamma_2$. At $\delta\gamma = 0$, the transparent DW case is recovered, see Sec. III A.

At arbitrary h , γ_1 , and γ_2 , Eq. (8) is solved numerically. A representative phase diagram is shown in Fig. 4. In contrast to the transparent case, the order parameter is now discontinuous across the DW, but it still is localized in its vicinity, see Figs. 5,6. The qualitative features of the phase diagram are quite similar to those discussed in Sec. III A. In particular, the crossover from the bulk regime to the strongly-localized superconducting DW regime is controlled by $\delta\gamma = \gamma_1 - \gamma_2$.

IV. NON-LINEAR EFFECTS

We have also studied the evolution of the shape of the superconducting nucleus as a function of temperature at zero external field, assuming a transparent DW with no local enhancement or suppression of superconductivity.

At $\gamma_1 = \gamma_2 = +\infty$, the order parameter and its derivative are continuous across the wall. Writing the order parameter in the dimensionless form: $\psi(\mathbf{r}) = \psi_0 e^{iqy} f(x)$, where $\psi_0 = \sqrt{K/\beta l_B^2}$, the free energy becomes

$$\frac{F}{F_0} = \left(\frac{df}{dx} \right)^2 + (\tau - 1)f^2 + (|x| + q)^2 f^2 + \frac{1}{2}f^4 \quad (12)$$

($F_0 = K^2/\beta l_B^4$), which yields the following non-linear GL equation:

$$-f'' + (|x| + q)^2 f + (\tau - 1)f + f^3 = 0 \quad (13)$$

The solution of the linearized version of this equation in Sec. III A gave the critical temperature $\tau_c \simeq 0.41$, corresponding to $q_* \simeq -0.77$.

The non-linear equation (13) is solved numerically at $0 < \tau < \tau_c$ by the “shooting” method [22]: We start at some large value of $|x| = x_0 \gg 1$ on both sides of the DW and then integrate the equation towards a matching point near the origin (although in the absence of an external field, the solution is symmetric about the DW, the matching point was not necessarily chosen to be at $x = 0$). For each τ and q , the starting value of $f(\pm x_0) = 0$ was chosen, while the starting slope of $f'(\pm x_0)$ was allowed to vary to find the solution, which is smooth everywhere. The upper and lower bounds on the initial slopes were estimated using the fact that at large $|x|$ the non-linear term in Eq. (13) is negligible, and $f(x)$ decays approximately as a Gaussian. Then the bisection method was used to converge on the starting slope that guaranteed continuity of the wave function. At the next step, for each wave function an evaluation of the GL free energy (12) was made, and the value of q was chosen to minimize F . The final results were found to be independent of the choice of the matching point.

Our results for the order parameter are presented in Fig. 6. The height of the wave function, i.e. $f(x=0)$,

grows approximately as $\sqrt{\tau_c(h=0)} - \tau$. As a check of the validity of our procedure, the temperature dependence of the parameter q was found to converge perfectly to the previously calculated value q_* as τ approaches $\tau_c(h=0)$.

V. CONCLUSIONS

We have studied the phase diagram of a superconducting domain wall in a ferromagnet using a phenomenological model featuring additional interface terms in the GL energy. These terms are introduced in order to account for the possible variation of the superconducting coupling

constant near the DW, and also for a finite transparency of the wall to electrons. The critical temperature for the localized DW superconductivity is found to be always higher than in the bulk. Depending on the relation between the phenomenological parameters in the interface terms, one can get an enhancement of the critical temperature above its value for a transparent wall, and also a non-monotonous dependence of the critical temperature on the external field.

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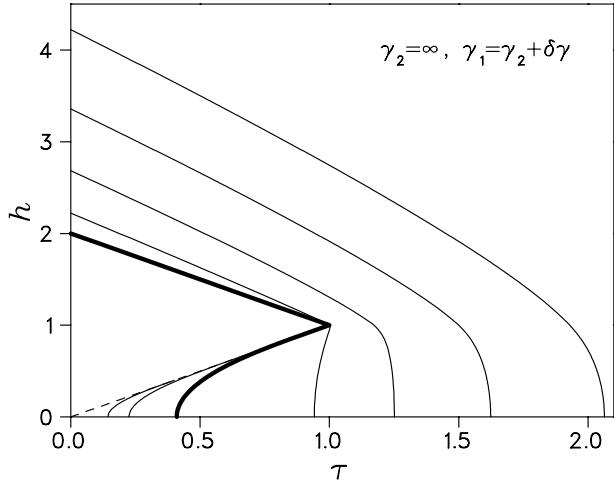


FIG. 1: The superconducting phase transition lines for a transparent DW with a local enhancement or suppression of superconductivity, i.e. $\gamma_1 = \gamma_2 + \delta\gamma$, $\gamma_{1,2} \rightarrow +\infty$. The dashed line corresponds to the bulk critical temperature $\tau_{c,bulk}(h) = 1 - |1 - h|$, the thick solid line – to a transparent DW case with $\delta\gamma = 0$, and the thin solid lines correspond to $\delta\gamma = 0.5, 0.3, -0.5, -0.7, -0.9, -1.1$ (from left to right).

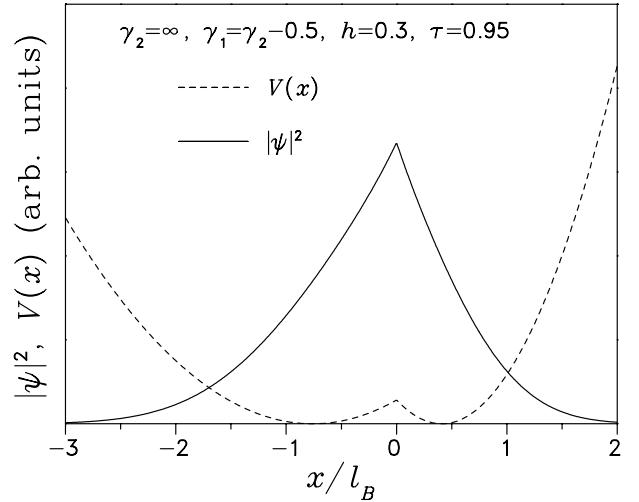


FIG. 2: The superconducting order parameter and the GL effective potential $V(x)$ for $h = 0.3$, at $\gamma_2 = \infty$, $\delta\gamma = -0.5$.

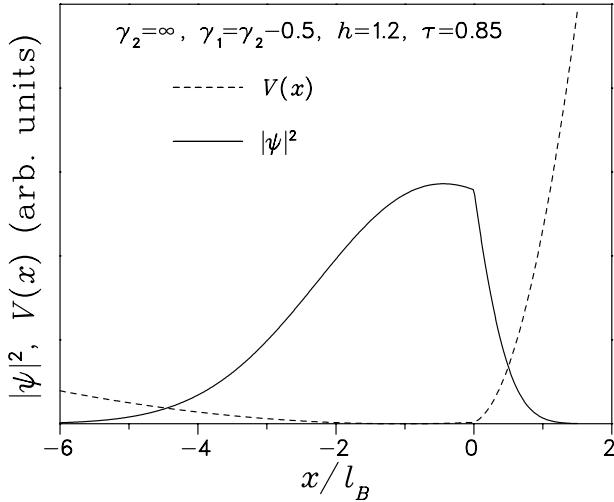


FIG. 3: The superconducting order parameter and the GL effective potential $V(x)$ for $h = 1.2$, at $\gamma_2 = \infty$, $\delta\gamma = -0.5$.

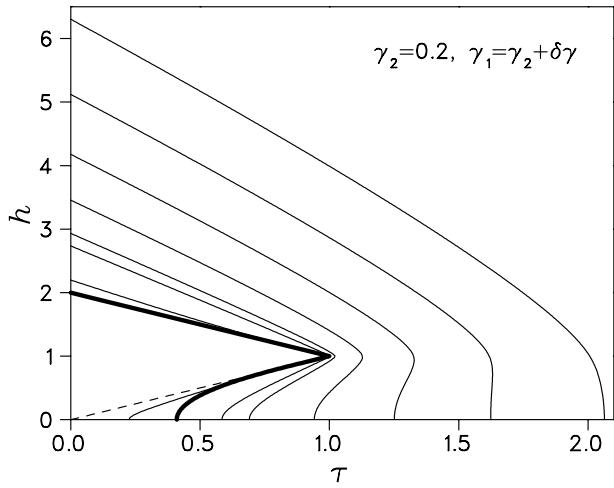


FIG. 4: The superconducting phase transition lines for $\gamma_1 = \gamma_2 + \delta\gamma$, $\gamma_2 = 0.2$. The dashed line corresponds to the bulk critical temperature, the thick solid line – to a transparent DW case with $\delta\gamma = 0$, and the thin solid lines correspond to $\delta\gamma = 0.5, 0, -0.1, -0.3, -0.5, -0.7, -0.9$ (from left to right).

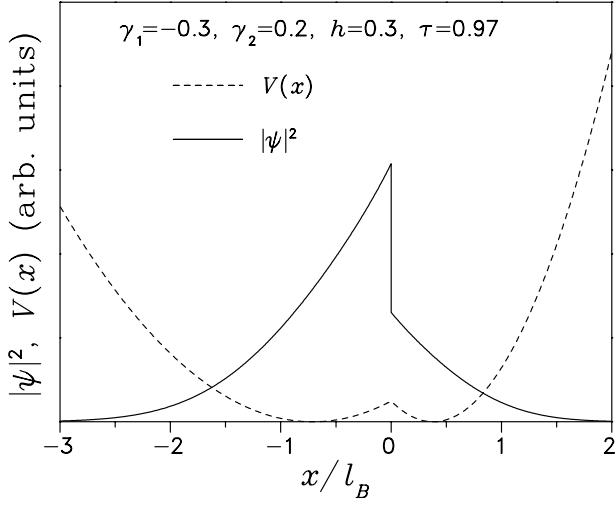


FIG. 5: The superconducting order parameter and the GL effective potential $V(x)$ for $h = 0.3$, at $\gamma_1 = -0.3, \gamma_2 = 0.2$.

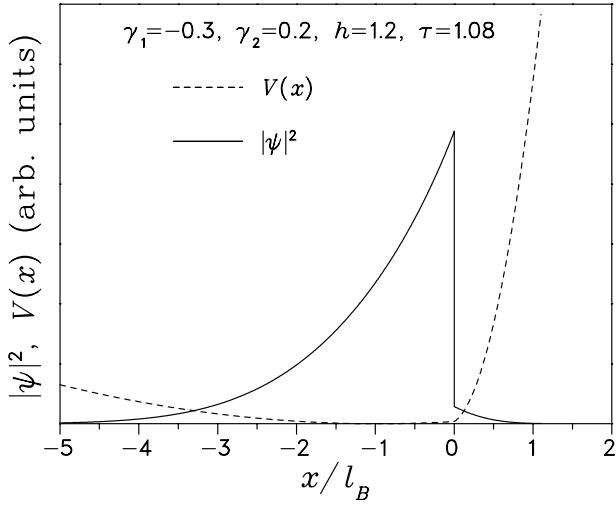


FIG. 6: The superconducting order parameter and the GL effective potential $V(x)$ for $h = 1.2$, at $\gamma_1 = -0.3, \gamma_2 = 0.2$.

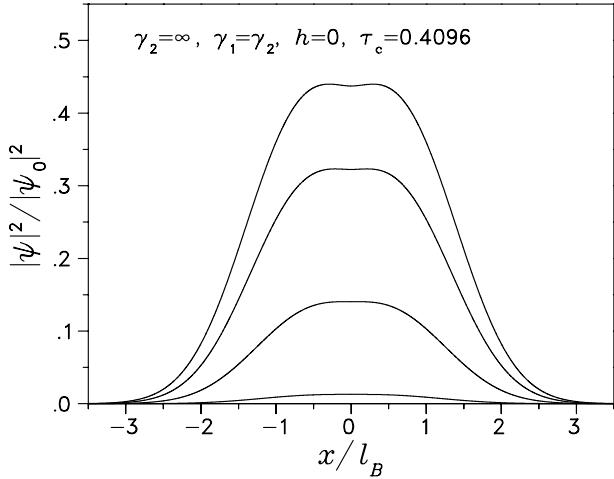


FIG. 7: The superconducting order parameter in the non-linear regime below τ_c , in the transparent DW case with no superconductivity enhancement [$\tau_c(h=0) \simeq 0.41$]. The curves correspond to $\tau = 0.40, 0.30, 0.15$, and 0.05 (from bottom to top).